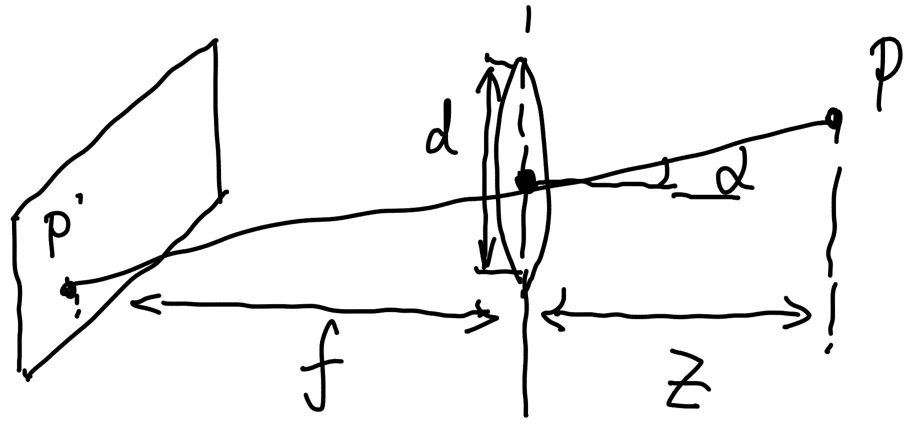
lect 5 Illumination & Photometric Stero

Irradiance



L: Radiance emitted from P to P'

E: Irradian on the film per square meter.

$$E = \left[\frac{\pi}{4} \frac{d^2}{f} \cdot \cos^4 \alpha \right] L$$

area of lens

Reflection:

1. Lambert's law.

$$B = \rho (N^T \cdot S)$$

Diffuse reflection

N: Normal Vector of the point

ρ: fraction of reflection radiance

S: light source vector.

B: power reflect

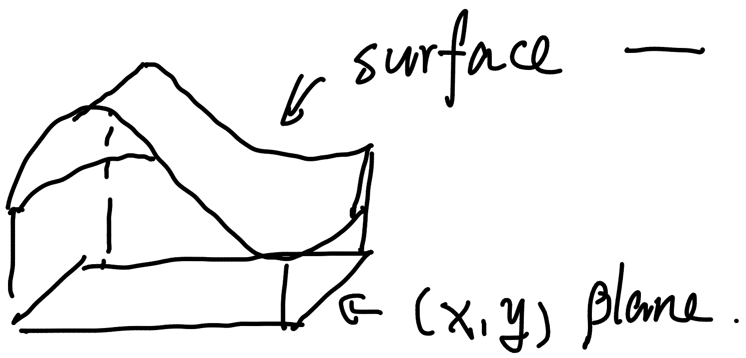
2. Specular Reflection

镜面.

Photometric Stero

Surface Model. Monge Patch

$f(x,y)$ is the height at location (x,y).



Target:

Get the $f(x,y)$ based on

$\begin{cases} \text{Light Source Vector} & S_j \\ \text{Image Pixel} & I_j(x,y) \end{cases}$

Step 1

Assumption.

Lambertion Model:

$$I_j(x,y) = \underset{\text{a scalar}}{k}\,\rho(x,y)\left(\underset{\text{Surface Norm}}{\underline{\overset{1\times3}{N(x,y)^T}}}\cdot \overset{3\times1}{S_j}\right)$$

$$= \underbrace{\rho(x,y)\cdot N(x,y)^T}\cdot \underbrace{k\cdot S_j}$$

$$= \underset{1\times3}{g(x,y)^T}\cdot \underset{3\times1}{V_j}$$

$g(x,y)^T$ is closely related to surface $f(x,y)$. has nothing to do with light. So. we have. n images of different light source.

$$\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix} = \begin{bmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{bmatrix} g(x,y)$$

$n\times 1$ $\qquad$ $n\times 3$ $\qquad$ 3×1

known

$$\Rightarrow \quad g(x,y) = \begin{bmatrix} V_1^T \\ \vdots \\ V_n^T \end{bmatrix}^{\dagger} \begin{bmatrix} I_1(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix}$$

Recall

$$g(x,y)^T = \underbrace{\rho(x,y)}_{\text{Scalar}} \cdot \underbrace{N(x,y)^T}_{\text{Unit Norm}}$$

$N(x,y)$ is a (3-1) Normal Vect.

It has 2-DoF.

$\rho(x,y)$ is a scalar. having 1-DoF

In all. there are 3-DoF.

So

$$\rho(x,y) = \|g(x,y)\|_2$$

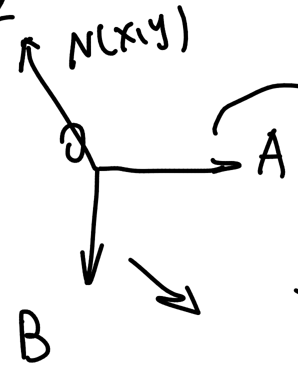
$$N(x,y) = \frac{g(x,y)}{\|g(x,y)\|_2}$$

Step 2.

We need to compute the surface $f(x,y)$

$$N(x,y) = \frac{1}{\sqrt{f_x^2+f_y^2+1}} \begin{bmatrix} -f_x \\ -f_y \\ 1 \end{bmatrix}$$

Proof



$f(x+dx, y) - f(x,y)$

$f(x, y+dy) - f(x,y)$

$$\vec{OA} = [dx,\ 0\ f_x]^T \qquad \vec{OB} = [0,\ dy \cdot f_y]^T$$

let $N(x,y) = [a, b, c]^T$

$\because N(x,y)^T \vec{OA} = N(x,y)\vec{OB} = 0$

$$\therefore \begin{cases} a\,dx + c f_x = 0 \\ b\,dy + c f_y = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{a}{c} = -\dfrac{f_x}{dx} \xrightarrow{dx \to 0} -\dfrac{\partial}{\partial x} f(x,y) \\ \dfrac{b}{c} = -\dfrac{f_y}{dy} \xrightarrow{dy \to 0} -\dfrac{\partial}{\partial y} f(x,y). \end{cases}$$

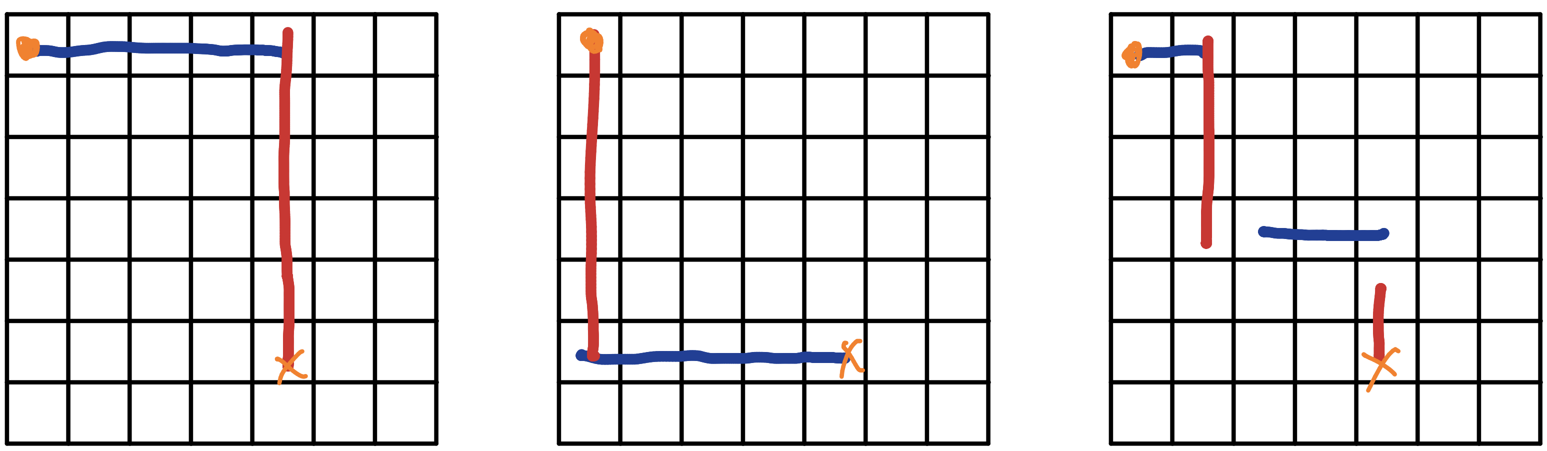
Step 3

Now, we know $f_x = -\frac{g^{(1)}(x,y)}{g^{(3)}(x,y)}$ $\quad f_y = -\frac{g^{(2)}(x,y)}{g^{(3)}(x,y)}$

we can use "Integration" to recover $f(x,y)$.

Technically, if $\frac{\partial^2}{\partial x \partial y} f(x,y) = \frac{\partial^2}{\partial y \partial x} f(x,y)$, then $f(x,y)$ is continuous. But, our recovered f_x, f_y may be prone to noise.

So, we may need to integrate via different paths and get the average



Three different paths.

Also, It's true that $f(x,y) = \iint f(x,y) + \underline{C}$, but because the surface is continuous, we don't really care the constant C.

If we know $N(x,y)$, Image Pixel. $I(x,y)$ can be

$$I(x,y) = N(x,y) \cdot S(x,y) + A$$

For points on the occluding contour,

$$\begin{bmatrix} I(x,y) \end{bmatrix} = \begin{bmatrix} N_x, & N_Y, & 1 \end{bmatrix} \begin{bmatrix} S_X \\ S_Y \\ A \end{bmatrix}$$ because $N_z = 0$

Limitations

① This model use orthographic camera. model, and assume lights are all parallel lights, ~~doesn't~~ doesn't consider projection

② Reflection model and light source model is too simple

③ No shadow

④ No Inter-reflection

⑤ No missing data

⑥ Integration path may affect the final result.